B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5DSE21

(Probability and Statistics)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Show that the standard deviation is independent of any change of origin but dependent on change of scale.

(b) Find the value of the constant k such that $f(x) = \begin{cases} kx^2(1-x) & \text{if } 0 < x < 1 \\ 0 & elsewhere \end{cases}$ is a probability density function.

(c) Find the moment generating function of Binomial distribution.

(d) Prove that $E(X^2) \geq \{E(X)\}^2$.

(e) Define random variable and distribution function.

(f) Let X be a random variable with $E(X) = 1, E[X(X-1)] = 4$. Find variance of $(7-2X)$.

(g) Write down the mean and variance of standard normal distribution.

(h) If X_n is a binomial (n, p) variate, then show that $\frac{X_n}{n} \underset{in\, p}{\longrightarrow} p$ as $n \to \infty$.

(i) Define moment generating function for a continuous random variable.

(j) Let X be a random variable. Show that the distribution function $F(x)$ of X is a monotonic non-decreasing function.

(k) State the Central limit theorem.

(l) Define Type I and Type II errors.

(m) Two random variables X and Y have zero means and standard deviations 1 and 2 respectively. Find the variance of $X + Y$ if X and Y are uncorrelated.

(n) Let the random variable X have a normal distribution. Does the random variable $Y = X^2$ also have a normal distribution?

(o) For a normal (m, σ) population, prove that $(\bar{X} - m)/\frac{\sigma}{\sqrt{n}}$ is normal (0, 1) variate.

2. Answer *any four* questions: 5×4=20

(a) If X is uniformly distributed in the interval (–1, 1), then find the distribution of $|X|$.

(b) If X is a Poisson variate with parameter μ, then show that $P(X \leq n) = \frac{1}{n!}\int_{\mu}^{\infty} e^{-x}x^n dx$, where n is any positive integer.

(c) If the correlation coefficient $\rho(X, Y)$ between two variables X and Y exists, then show that $-1 \leq \rho(X, Y) \leq 1$.

(d) A random variable X has the probability mass function $f(x) = \frac{1}{2x}, x = 1, 2, 3, ...$

Find its (i) moment generating function, (ii) mean and (iii) variance.

(e) What is sampling distribution of a statistic? Show that the function $t = \frac{\sqrt{\mu}(X-m)}{s}$ is a t-distribution with $n - 1$ degrees of freedom. 1+4

(f) Prove that the maximum likelihood estimate of the parameter α of the population having density function $f(x) = \frac{2(\alpha - x)}{\alpha^2}, (0 < x < \alpha)$, for a sample x_1 of unit size is $2x_1$ and this estimate is biased.

3. Answer *any two* questions: 10×2=20

(a) (i) If X and Y are two independent normal variates (m_X, σ_X) and (m_Y, σ_Y) respectively, then show that $U = X + Y$ is a normal variate (m, σ) where $m = m_X + m_Y$ and $\sigma^2 = \sigma_X^2 + \sigma_Y^2$.

(ii) If the random variable X is uniformly distributed over $(-2, 2)$, then find the mean and variance of the random variable $\min\{X, 1\}$. 5+5

(b) The joint probability density function of two random variables X and Y is $k(1 - x - y)$ inside the triangle formed by the axes and the line $x + y = 1$ and zero elsewhere. Find the value of k and $P\left(X < \frac{1}{2}, Y > \frac{1}{4}\right)$. Find also the marginal distributions of X, Y and determine whether the random variables are independent or not. 2+2+2+2+2

(c) (i) State Tchebycheff's inequality and weak law of large numbers.

(ii) The distribution of a random variable X is given by $P(X = -1) = \frac{1}{8}$, $P(X = 0) = \frac{3}{4}$, $P(X = 1) = \frac{1}{8}$. Verify Tchebycheff's inequality for the distribution. 2+2+6

(d) (i) Find the expectation of the sum of points on n unbiased dice.

(ii) If $X_1, X_2, \ldots X_n, \ldots$ be a sequence of random variables such that $S_n = X_1 + X_2 + \cdots X_n$ has a finite mean M_n and standard deviation Σ_n for all n. If $\frac{\Sigma_n}{n} \to 0$ as $n \to \infty$, then show that $\frac{S_n - M_n}{n} \xrightarrow[in\ P]{} 0$ as $n \to \infty$.

(iii) Prove that sample mean is unbiased estimate of the corresponding population mean provided the population mean exists. 4+3+3

B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5DSE22

(Portfolio Optimization)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20
   - (a) What do you mean by cash flow?
   - (b) Describe the two well known risk premiums.
   - (c) Define return relative.
   - (d) What are the properties of normal distribution?
   - (e) What is a growing annuity?
   - (f) Discuss the important turnover ratio.
   - (g) What are the different types of financial ratios?
   - (h) Explain the important profit margin ratio.
   - (i) Discuss the key valuation ratio.
   - (j) What is the risk of a 2-security portfolio?
   - (k) What is an efficient portfolio?
   - (l) State the relationship between covariance and correlation.
   - (m) What is the expected return on a portfolio of risky assets?
   - (n) Discuss the limitations of betas based on accounting earning.
   - (o) What adjustment is done to historical betas?

2. Answer *any four* questions: 5×4=20
   - (a) What is a multifactor model? Describe the types of multifactor models used in practice. 2+3
   - (b) Discuss the procedure commonly used in practice to test the CAMP.
   - (c) Explain the nature of a risk-return indifference curve.
   - (d) What is the risk-free rate? How would you measure it? 3+2
   - (e) Explain the single index model proposed by William Sharpe.
   - (f) Describe the procedure developed by Markowitz for choosing the optimal portfolio of risky assets.

3. Answer *any two* questions: 10×2=20

(a) Define the return generating process according to APT. What is the equilibrium risk return relationship according to APT? 5+5

(b) The stock of ABC Limited performs well relative to other stocks during recessionary periods. The stock of XYZ Limited, on the other hand, does well during growth periods. Both the stocks are currently selling for Rs. 100 per share. Assess the rupee return (dividend plus price) of these stocks for the next year as follows:

	Economic Condition		Stagnation	Recession
	High Growth	Low Growth		
Probability	0·3	0·4	0·2	0·1
Return on ABC's stock	100	110	120	140
Return on XYZ's stock	150	130	90	60

Calculate the expected return and standard deviation of investment: 3+3+4

(i) Rs. 1,000 in the equity stock of ABC Limited

(ii) Rs. 1,000 in the equity stock of XYZ Limited

(iii) Rs. 500 each in the equity stock of ABC Limited and XYZ Limited.

(c) (i) What is an annuity? What is the difference between an ordinary annuity and an annuity due?

(ii) What is the present value of the following cash flow stream if the discount rate is 14 per cent? (2+3)+5

Year	0	1	2	3	4
Cash flow	5000	6000	8000	9000	8000

(d) (i) What is the risk of an *n*-security portfolio?

(ii) Show why the covariance term dominates the risk of a portfolio as the number of securities increase.

(iii) How does the efficient frontier change, when the probability of lending and borrowing at a risk-free rate is introduced? 2+5+3

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B.A/B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5DSE23

(Boolean Algebra and Automata Theory)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Let L be a lattice and $a, b, \in L$. Then show that $a \leq b$ implies $a \vee c \leq b \vee c$.

(b) Determine the minterm normal form of the Boolean expression $f(x_1, x_2) = x_1 \vee x_2$.

(c) Define a recursive language and give an example.

(d) Construct a finite automata equivalent to the regular expression: $L = a\,a^*\,(a + b)^*$.

(e) Give an example of a lattice where distributive laws do not hold.

(f) State the recognition problem.

(g) Construct a logic circuit that produces $(x + y + z)(xyz)'$ as its output.

(h) What is pushdown automata? Give an example.

(i) Give an example of Turing Machine that accepts the 'Empty Language'.

(j) Draw a transition diagram for a machine that recognizes whether or not a string in B^* contains an even number of 1's.

(k) What is NFA?

(l) What is ambiguous grammar?

(m) Define down-sets and give an example.

(n) Define a string and give an example.

(o) Find the simplified form of the Boolean function: $a + a\bar{b}$.

2. Answer *any four* questions: 5×4=20

(a) Show that every regular language is a content-free language.

(b) Show that the set of Turing-machine codes for Turing Machines that accept all inputs that are palindromes (possibly along with some other inputs) is undecidable.

(c) Construct a PDA to accept the following language: $L = \{a^n b^{2n} | n \geq 1\}$.

(d) Transform the following DNF to CNF:

$$x_1'\,x_2'\,x_3' + x_1'\,x_2'\,x_3 + x_1\,x_2'\,x_3' + x_1\,x_2\,x_3'$$

(e) Design a Turing Machine that computes the function $f(x,y) = x + y$ if $x \geq y$
$= 0$ if $x < y$.

(f) Show that in a complemented distributive lattice the following are equivalent:

(i) $a \leq b$

(ii) $a \wedge b' = 0$

(iii) $a' \vee b = 1$

(iv) $b' \leq a'$

3. Answer *any two* questions: 10×2=20

(a) (i) Design a Turing machine that accepts the language $L = \{0^{2^n} | n \geq 0\}$.

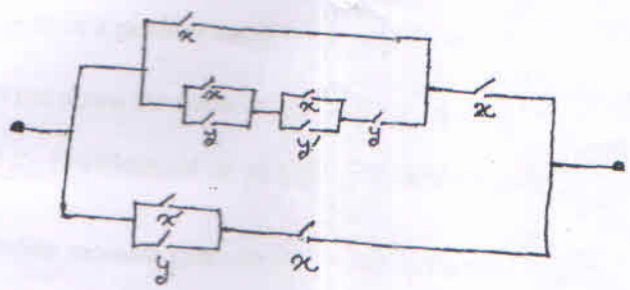
(ii) Prove that for any transition function δ and for any two input strings x and y $\delta(q, xy) = \delta(\delta(q, x), y)$. 6+4

(b) (i) Suppose a 3-variable Boolean term is given as follows:
$\emptyset = xy + xz' + yz$.
Minimize ϕ using K-map.

(ii) Suppose $\emptyset(a, b, c, d) = \Sigma m(0,1,3,7,8,9,11,15)$. Minimize ϕ using Quine-McCluskey method. 5+5

(c) (i) Prove that a lattice L is distributive if and only if $x \wedge (y \vee z) \leq (x \wedge y) \vee z$, for all $x, y, z, \in L$.

(ii) Obtain a Boolean expression which represents the following circuit. Moreover, draw an equivalent circuit as simple as you can: 5+5



(d) Discuss about the equivalences of deterministic and non-deterministic automata and find the equivalent deterministic finite automata for the given non-deterministic finite automata: 5+5